

Lecture 1a: Introduction to Axions

Cem Eröncel

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In this lecture, we will provide a short introduction to the Strong CP problem and axions. This can be seen as a brief summary of what we will be covering during the course.

1 The Standard Model and its Problems

It is fair to claim that the Standard Model of Particle Physics (SM) is one of the triumphs of Theoretical High Energy Physics in the 20th century. It provides the most accurate and complete description of fundamental particles and the interactions between them. The discovery of the Higgs boson in July 2012 at CERN¹ did provide the last remaining piece, so that the SM has been verified up to the energy scale and experimental accuracy that today's collider technology can reach.

Despite of this success, lots of hints suggest that the SM is only an *effective* theory up to some unknown cutoff scale, and there should exist new physics *beyond the Standard Model*, BSM for short. A partial list with brief explanations can be given as follows:

- **Baryon asymmetry:** Our observable universe contains vastly more matter than antimatter. But, there is no reason that the universe should start with more matter. Thus, there should be a process responsible generating the asymmetry. Such a process is not possible with the ingredients of SM.
- **Hierarchy problem:** In the SM, the Higgs mass receives quantum corrections on the order of the cutoff scale Λ . Experiments show that the cutoff scale should be larger than TeV so it is a mystery why the Higgs mass $m_H \approx 126 \text{ GeV}$ can remain so small.
- **Neutrino masses:** Neutrinos are massless particles in the SM. However, we learned from the neutrino oscillations experiments that they *do* have a tiny but non-zero mass.
- **Quantum gravity:** The SM provides a quantum description for the three of the four fundamental forces. These are electromagnetic, weak, and strong interactions. The remaining force, gravity, cannot be explained in the framework of SM.

¹ Georges Aad et al. (2012). In: *Phys. Lett. B* 716, pp. 1–29. arXiv: [1207.7214 \[hep-ex\]](#); Serguei Chatrchyan et al. (2012). In: *Phys. Lett. B* 716, pp. 30–61. arXiv: [1207.7235 \[hep-ex\]](#).

- **Dark energy:** From the supernova observations we learned that the universe is not only expanding but the expansion is accelerating. The mysterious “energy” responsible for accelerated expansion cannot be provided by the SM².
- **Dark matter:** The latest observations of the Cosmic Microwave Background (CMB) show that the SM fields can only provide 15% of the matter in the observable universe. The rest of it consists of a hypothetical substance called *dark matter*. If it is a particle, then it should be electrically neutral, weakly interacting, and stable. The SM does not have such a candidate.
- **Strong CP problem:** In simple terms, this problem stems from the question why the CP invariance is broken by the weak interactions, but not by the strong interactions.

In this course we will see that the hypothetical *axion* particle can solve the Strong CP problem, and at the same time it can account all or part of dark matter. This makes the axion one of the best motivated candidates for BSM physics.

2 Strong CP Problem

They are three important transformations in the Quantum Field Theory that are not connected to the identity. These are

- **C for charge conjugation:** Transforms a particle to its antiparticle and vice versa.
- **P for parity:** Flips the sign of the spatial coordinate, i.e. $P : (t, \mathbf{x}) \mapsto (t, -\mathbf{x})$.
- **T for time reversal:** As its name suggest, flips the sign of the time coordinate, i.e. $T : (t, \mathbf{x}) \mapsto (-t, \mathbf{x})$.

The celebrated **CPT Theorem**³ states that the Lorentz invariance and unitarity requires that all the terms in the Lagrangian should be invariant under the simultaneous application of C, P and T. However, the Lagrangian, hence the physics, might not be invariant under separate applications of these transformations. For example, the parity is violated in weak interactions since left-handed and right-handed fermions interact differently⁴. It has also been confirmed experimentally that the combination of C and P, named CP, is violated by the weak interactions⁵ which lead to 1980 Nobel prize.

Since the CP symmetry is already broken by the weak interactions, we might expect that it is broken by the strong interactions as well. In

² A naive derivation leads to a 120 orders of magnitude discrepancy between the theoretical prediction and the observation, famously known as the worst prediction in the history of physics. This is known as the *cosmological constant problem*.

³ R. F. Streater and A. S. Wightman (1989). *PCT, spin and statistics, and all that*. Princeton, NJ: Princeton University Press.

⁴ A parity transformation turns left-handed spinors to right-handed spinors and vice versa.

⁵ J. H. Christenson et al. (July 1964). In: *Phys. Rev. Lett.* 13 (4), pp. 138–140.

fact, as we will see in the coming lectures, the Quantum Chromodynamics (QCD) Lagrangian contains a CP-violating term

$$\mathcal{L}_{\text{QCD}} \supset \frac{\theta g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a \quad (1)$$

which is known as the θ -term. Here g_s is the strong gauge coupling, $G_{\mu\nu}^a$ is the gluon field strength with the label a and $\tilde{G}_{\mu\nu}^a$ is the dual field strength defined by

$$\tilde{G}_{\mu\nu}^a = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a\rho\sigma}, \quad \text{with } \epsilon^{0123} = -1. \quad (2)$$

Due to this term, the neutron gets a non-zero electric dipole moment⁶

$$d_n = 1.2 (0.5) \times 10^{-2} \theta e \text{GeV}^{-1}. \quad (3)$$

However, the neutron electric dipole moment has not been observed so far with the current upper bound⁷

$$|d_n| < 1.8 \times 10^{-26} e \text{cm} \approx 0.9 \times 10^{-12} e \text{GeV}^{-1} \quad (90\% \text{ C.L.}) \quad (4)$$

This yields an upper bound for the θ -parameter:

$$|\theta| \lesssim 10^{-10}. \quad (5)$$

This tells us that the θ -parameter of the QCD should be very tiny if not zero. Why this value is so small is the **Strong CP problem**.

At this point it will be useful to present a conceptual picture of the Strong CP problem which is due to Anson Hook⁸ since it will help us to understand the axion solution very easily. Let us remember that the electric dipole moment simply measures the separation of positive and negative electrical charges in a system. The neutron consists of one up quark and two down quarks. The up quark has electric charge $+2/3$ which down quark has $-1/3$. Thus, the neutron is electrically neutral. The electric dipole moment is determined by the distribution of the three quarks inside the neutron. A schematic picture is shown in Figure 1. Note that for $\theta \neq 0$, the charge distribution is asymmetric, thus the neutron will have a non-zero electric dipole moment. Unobservation of this moment tells us that the quarks inside the neutron should be “aligned” like shown in Figure 2.

3 Axion solution to the Strong CP problem

With the discussion above, it is fairly easy to make sense of the axion solution to the Strong CP problem, at least conceptually. Observe that in this picture, the Strong CP problem is really the question why the quarks inside the neutron are aligned as in Figure 2 but not as in Figure

⁶ Maxim Pospelov and Adam Ritz (2000). In: *Nucl. Phys. B* 573, pp. 177–200. arXiv: [hep-ph/9908508](#).

⁷ C. Abel et al. (2020). In: *Phys. Rev. Lett.* 124.8, p. 081803. arXiv: [2001.11966 \[hep-ex\]](#).

⁸ Anson Hook (2019). In: *PoS TASI2018*, p. 004. arXiv: [1812.02669 \[hep-ph\]](#).

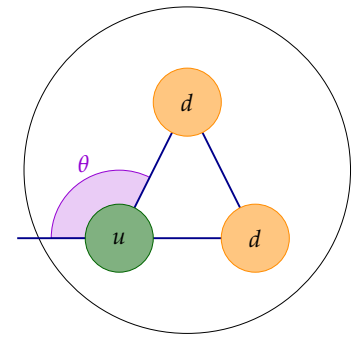
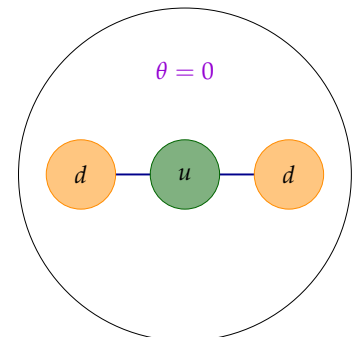
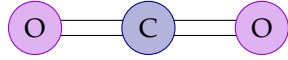


Figure 1: A schematic picture of the distribution of quarks inside the neutron.



1. These kind of symmetric arrangements exists in nature, for example consider the CO₂ molecule



which also has zero electric dipole moment thanks to its symmetric configuration. In the CO₂, this arrangement is the *lowest energy* configuration so if the system is disturbed away, it will relax back *dynamically*. Thus, the vanishing of the CO₂ electric dipole moment is not mysterious at all.

The situation is different for the neutron since θ is just a parameter in the SM, not a dynamical variable. But we can assume that there exists a theory beyond the SM in which the θ parameter is promoted dynamical variable, namely to a field $\theta \rightarrow \theta(x)$. Furthermore, we can assume that there exists a potential $V(\theta)$ that is minimized at $\theta = 0$. In such a theory, $\theta = 0$ is simply explained by the system dynamics like in the CO₂ case. This is precisely what happens in the **Peccei-Quinn (PQ)** solution of the Strong CP problem.⁹

Like any field, $\theta(x)$ will also have excitations around the minimum that correspond to particles. These particles are called **axions**.¹⁰ The name is given by Frank Wilczek who is inspired by a laundry detergent with the same name.¹¹ In the coming lectures, we will calculate the axions mass using *Chiral Perturbation Theory* as

$$m_a \simeq 5.7 \left(\frac{10^{12} \text{ GeV}}{f_a} \right) \mu\text{eV}, \quad (6)$$

where f_a is called the **axion decay constant** which is roughly the scale of new physics that has generated the axion. Many experiments suggest that f_a should be larger than 10^8 GeV which means that the axion should be very light.

4 Axion Electrodynamics

Recall that the Maxwell's equations can be derived from the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (7)$$

As we will see later in the course, the axion necessarily couples to the electromagnetism via the interaction term

$$\mathcal{L} \supset \frac{1}{4} g_{a\gamma\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad g_{a\gamma\gamma} = \mathcal{O}(1) \times \frac{\alpha_{\text{em}}}{2\pi f_a}, \quad (8)$$

where $\alpha_{\text{em}} \approx 1/137$ is the fine structure constant, and $\phi = \theta f_a$. So the axion, if it exists, modifies the Maxwell's equations. The electrodynamics together with an axion is called *Axion Electrodynamics*. Utilizing

⁹ R. D. Peccei and Helen R. Quinn (1977). In: *Phys. Rev. Lett.* 38, pp. 1440–1443.

¹⁰ Steven Weinberg (1978). In: *Phys. Rev. Lett.* 40, pp. 223–226; Frank Wilczek (1978). In: *Phys. Rev. Lett.* 40, pp. 279–282.

¹¹ Frank Wilczek (2023). *Time's (Almost) Reversible Arrow*. Quanta Magazine. (Visited on 02/20/2023).

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